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We consider the evolution of primordial black holes formed during the high energy phase of the braneworld scenario. By assuming that a small fraction of the total energy is initially trapped in black holes, we show that the effect of accretion from the surrounding radiation bath is dominant compared to evaporation for such black holes. This feature lasts till the onset of matter (or black hole) domination resulting in the net growth of mass of black holes. We calculate the consequent enhancement of the black hole lifetimes in two different scenarios depending upon whether radiation domination persists throughout the high energy phase or not. We find that the black hole evaporation times are significantly large for even very small black holes which could survive till the present era for a range of the parameters.

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The braneworld scenario in which our observable universe is a brane embedded in a higher dimensional bulk has gained much popularity in recent times. The motivation for such a consideration originates from solutions of string theory where matter and radiation are confined to the 3-brane, whereas gravity propagates also in the bulk. The significant consequences of this idea on the physics of the early universe is the focus of much current attention [1]. The foremost departure from standard cosmology is that there exists a regime during the early stages when the expansion rate of the universe is proportional to its energy density. The aim is to look for observable implications of this behaviour on cosmolgy.

Primordial black holes through their evolution and evaporation are potentially interesting candidates towards probing the high energy early stages of brane cosmology. The formation of primordial black holes through various mechanisms has been studied in much detail [2]. More recently, the diverse cosmological ramifications of primordial black holes such as their impact on baryogenesis, nucleosynthesis, quintessence, reionization, gamma rays, neutrino background spectra, and dark matter are being vigorously invesigated [3,4]. Most of these studies have been performed in the context of standard cosmology. Recently, several attempts of describing the formation, evaporation and cosmological consesquences of primordial black holes in the braneworld scenario have been undertaken [5].

In the present Letter, we consider the evolution of primordial black holes formed during the high energy brane phase when the expansion rate of the universe is linearly proportional to its energy density [1]. We restrict ourselves to the Randall-Sundrum Type II model [6]. Black holes formed during this regime evaporate at a rate proportional to their effective area times the fourth power of their temperature. Using the induced metric on the brane, and expressions for the black hole radius and temperature in terms of the AdS radius $l$, Guedens et al [7] derived an evaporation law which is different from that of usual $4D$ black holes. However, for a more complete analysis of the evolution of black holes, one must take into account the effect of accretion from the surrounding radiation bath as well. This point has been emphasized by Zeldovich and Novikov [2] and the ramifications of considering accreting black holes have been found to be significant in the context of standard cosmology [4]. In our analysis, we consider the effect of both accretion and evaporation on primordial black holes formed on the brane.

To begin with, let us consider the rate of change of mass $\dot{M}$ of a black hole immersed in a radiation bath. The accretion rate is proportional to the surface area of the black hole times the energy density of radiation. The evaporation rate, as mentioned earlier, is proportional to the surface area times the fourth power of temperature. Taking into account both these effects together, $\dot{M}$ is given by

$$\dot{M} = 4\pi r_{BH}^2 \left(-g_{brane}\sigma T_{BH}^4 + \rho_R\right) \tag{1}$$

where $r_{BH}$ and $T_{BH}$ are the radius and temperature of the black hole, $\sigma$ is the Stefan-Boltzmann constant, and $\rho_R$ is the energy density of radiation given by

$$\rho_R = \frac{3M_4^2}{32\pi t_c t} \tag{2}$$

$$r_{BH} = \left(\frac{8}{3\pi}\right)^{1/2}\left(\frac{l}{l_4}\right)^{1/2}\left(\frac{M}{M_4}\right)^{1/2} l_4 \tag{3}$$

$$T_{BH} = \frac{1}{2\pi r_{BH}} \tag{4}$$

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$$\sigma = \frac{\Gamma(4)\zeta(4)}{8(\pi)^2} \tag{5}$$

where ($t_c \equiv l/2$) is the transition time to standard cosmology, and the above relations hold for $t << t_c$. (Note here that we have neglected the contribution of evaporation into the bulk given by a term proportional to $4\pi r_{BH}^2 g_{bulk} T_{BH}^5$ since it is subdominant even for very small black holes [7]). With the above substitutions the black hole equation can be written as

$$\dot{M} = -\frac{AM_4^2}{Mt_c} + \frac{BM}{t} \tag{6}$$

where $A$ and $B$ are dimensionless numbers given by [1]

$$A \simeq \frac{3}{(16)^3\pi} \tag{7}$$

$$B \simeq \frac{2}{\pi} \tag{8}$$

The exact solution for the black hole equation is given by

$$M(t) = \left[\left(M_0^2 - \frac{2AM_4^2}{2B-1}\frac{t_0}{t_c}\right)\left(\frac{t}{t_0}\right)^{2B} + \frac{2AM_4^2}{2B-1}\frac{t}{t_c}\right]^{1/2} \tag{9}$$

with $M = M_0$ at the time of BH formation $t = t_0$. It can be seen that an extremum for the function $M(t)$ exists only if $(M_0/M_4)^2 < (2A/2B-1)(t_0/t_c)$. However, using the relation between formation time and mass [7]

$$\frac{t_0}{t_4} \simeq \frac{1}{4}\left(\frac{M_0}{M_4}\right)^{1/2}\left(\frac{l}{l_4}\right)^{1/2} \tag{10}$$

one can see that such a small initial mass will violate the lower bound on horizon mass $(M_H/M_4) > 2\times10^6(l_4/l)^{1/3}$ obtained from the contribution of gravitational waves towards CMBR anisotropy [8]. This result shows that for a radiation dominated high energy phase ($t < t_c$), accretion dominates over evaporation for all times and the mass of a primordial black hole continues to grow monotonically with

$$\frac{M(t)}{M_0} \simeq \left(\frac{t}{t_0}\right)^B \tag{11}$$

We have so far taken into account the case of a single black hole immersed in radiation. However, from a cosmological point of view one needs to consider a scenario in which a certain number density of primordial black holes exchange energy with the surrounding radiation by accretion and evaporation. Black holes are formed with an initial mass spectrum whose range is much debated [9]. In order to keep our analysis uncomplicated, the subsequent calculations are performed by considering an average initial mass $M_0$. We thus assume that at a time $t_0$ the fraction of the total energy in black holes is $\beta$, and the number density of black holes is $n_{BH}(t_0)$. Hence, we have

$$\rho_T(t_0) = \rho_R(t_0) + \rho_{BH}(t_0) \tag{12}$$

$$\rho_{BH}(t_0) = \beta\rho_T(t_0) = M_0 n_{BH}(t_0) \tag{13}$$

$$\rho_R(t_0) = (1-\beta)\rho_T(t_0) \tag{14}$$

The number density of black holes $n_{BH}(t)$ scales as $a(t)^{-3}$, and thus for a radiation dominated evolution on the brane, one has $(n_{BH}(t)/n_{BH}(t_0)) = (t_0/t)^{3/4}$ since $a(t) \propto t^{1/4}$. The net energy in black holes grows since accretion dominates over evaporation. The condition for the universe to remain radiation dominated (i.e., $\rho_{BH}(t) < \rho_R(t)$) at any instant $t$ can be derived from Eqs.(9,12,13,14) to be

$$\beta < \frac{(t_0/t)^{B+1/4}}{1 + (t_0/t)^{B+1/4}} \tag{15}$$

Thus depending on the value of the parameter $\beta$ there may or may not be an era of black hole domination in the early brane dominated case. We shall analyse these two cases separately.

Let us first consider the situation when the cosmology stays radiation dominated up to the time when brane effects are important, i.e., $t \leq t_c$. For this to be the case, the initial fraction $\beta$ should be such that

$$\frac{\beta}{1-\beta} < \left(\frac{t_0}{t_c}\right)^{B+1/4} \tag{16}$$

If the universe remains radiation dominated up to time $t_c$, then (for $M_0 \geq M_4$) the mass of a black hole is given by Eq.(11). However, the black hole must remain small enough, i.e.,

$$(M/M_4) < (3\pi/4)(t/t_4) \tag{17}$$

for it to obey the $5D$ evaporation law that we have used [7]. These criteria can be used to put an upper bound on the initial mass:

[1] We assume that the black holes can emit massless particles only and take $g_{brane} = 7.25$ as in [7]. It is known that the cross section for particle absorbtion is increased for relativistic particles [2], and also for relativistic motion of black holes [4]. In the present analysis we neglect these effects which tend to increase the value of $B$ by a small amount.

$$\frac{M_0}{M_4} < \left(\frac{3\pi}{4(2\sqrt{2})^B}\right)^{\frac{2}{2-B}} \frac{t_c}{t_4} \tag{18}$$

In the low energy regime (when $t > t_c$) one has $\rho_R \propto t^{-2}$. The black hole mass will continue to grow due to accretion up to a certain time (say $t_t$) after which the radiation density becomes too dilute for accretion to be significant. At this stage the rate of evaporation is also insignificant since the black hole masses have grown by several orders of magnitude from their initial values. So there ensues an era during which the black hole mass stays nearly constant over a period of time until evaporation takes over finally. This feature is well supported by numerical simulations of coupled black hole and Einstein equations for $4D$ black holes [4]. For the present case the accretion rate is smaller since the surface area is $\propto M$ instead of $M^2$ for $4D$ black holes. Furthermore, the evaporation rate is ($\propto M^{-1}$) instead of $M^{-2}$. Hence, a critical mass $M_{max}$ is reached before evaporation starts dominating from the transition time $t_t$ onwards[2]. Assuming Eq.(17) to be still valid together with radiation domination at the time $t_t$, the lifetime for a typical black hole can be computed from Eq.(6) (using $M = M_{max}$ at $t = t_t$) and is given by

$$\frac{t_{end}}{t_4} \simeq \frac{1}{2A}\left(\frac{M_{max}}{M_4}\right)^2 \frac{t_c}{t_4} \tag{19}$$

Using Eqs.(10) and (11), the lifetime is given by

$$\frac{t_{end}}{t_4} \simeq \frac{4}{A}(2\sqrt{2})^B \left(\frac{M_0}{M_4}\right)^{2-B} \frac{t_c}{t_4}\left(\frac{t_t^2}{t_c t_4}\right)^B \tag{20}$$

Guedens et al [7] derived the increase of black hole evaporation time compared to $4D$ black holes as $\propto (l/r_0)^2$, i.e.,

$$\frac{t_{end}(M_0, 5D)}{t_{end}(M_0, 4D)} \simeq \frac{t_c}{t_4}\frac{M_4}{M_0} \tag{21}$$

Our results show that over and above the enhancement of black hole lifetime originating from a modified evaporation law, the black hole lifetime is further increased by a factor $\propto (M_{max}/M_0)^2$ purely due to accretion given by

$$\frac{t_{end}(M, 5D)}{t_{end}(M_0, 5D)} \simeq \left(\frac{M_4}{M_0}\right)^B \left(\frac{t_t^2}{t_c t_4}\right)^B \tag{22}$$

for $t_c \leq t_t \leq t_{eq}$. We thus find that the effect of accretion in prolonging the life time of blackholes is more pronounced compared to the $5D$ evaporation effect for small black holes. From Eq.(20) it can be seen that for $(l/l_4) \sim 10^{20}$, a small number density of primordial $5D$ black holes formed with very low or even subplanckian masees ($M_0 \leq M_4$) could survive up to the era of nucleosynthesis and beyond if $t_t \geq 10^5 t_c$. This result is likely to have multifarous cosmological consequences [3] which need to be investigated in detail. But for black holes with large initial masses ($M_0 >> M_4$) which form later (from Eq.(10)) one sees that because of a lower total mass gain up to $t_t$, the black hole evaporation time is not increased by much over and above the factor arising due to the $5D$ evaporation law. Hence, for example, if we take $t_t \sim 10^5 t_c$, and $(l/l_4) \sim 10^{20}$, black holes formed with $M_0 = 10^{15} M_4 \simeq 10^{10}$g, will have $M_{max} \simeq 10^{15}$g, and will evaporate now completing their life cycle as $5D$ black holes. However, if the AdS radius is the maximum allowed by the current experimental bounds [10] coming from corrections to the Newtonian potential of a point mass [6], i.e., $(l/l_4) \simeq 10^{30}$ one can verify that even for $t_t \sim t_c$, black holes with initial masses as low as $M_0 = 10^8 M_4 \simeq 10^3$g survive up to the present era.

Let us now consider the case when the initial energy fraction $\beta$ is such that the the accretion process causes $\rho_{BH}(t)$ to exceed $\rho_R(t)$ at a time $t < t_c$. If $\beta$ exceeds the bound (we still assume $\beta << O(1)$) given in Eq.(15), then the cosmology enters a matter (black hole) dominated regime in the high energy phase when brane corrections to the Freidmann equations are still important, i.e., $H^2 \propto \rho^2$. The onset of such a matter dominated era is labelled by $t_{heq}$ which can be written from Eqs.(11-14) as[3]

$$\frac{t_{heq}}{t_0} = \left(\frac{1-\beta}{\beta}\right)^{\frac{4}{4B+1}} \equiv \gamma \tag{23}$$

The mass of a black hole at $t_{heq}$ is given by $M(t_{heq}/M_0) = \gamma^B$.

For $t > t_{heq}$ the Hubble expansion is essentially driven by the black holes ($p = 0$) which dominate over radiation. Since the number density of black holes scales as matter ($n_{BH}(t) \propto a^{-3}$), for $t < t_c$ one has $H \propto \rho_{BH}$, and thus the scale factor grows as $a \sim t^{1/3}$. During this era, the radiation density $\rho_R$ is governed by the equation

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[2]The exact determination of $t_t$ requires the numerical integration of the coupled Einstein and black hole equations (see, for example [4]) and is beyond the scope of the present paper.

[3]It should be kept in mind that although the formation time $t_0$ of individual black holes is spread out in accordance with the initial mass spectra, or vice-versa, we have used $t_0$ to signify the mean formation time in the similar sense of using $M_0$ as the mean initial mass.

$$\frac{d}{dt}\left(\rho_R(t)a^4(t)\right) = -\dot{M}(t)n_{BH}(t)a(t) \qquad (24)$$

where the r.h.s (contribution from accretion black holes) should not be neglected in comparison with the normal redshiting term ($\rho_R \sim a^{-4}$) because at this stage the black holes dominate the total energy density. Taking into account the radiation dominated expansion for $t < t_{heq}$, and black hole dominated expansion for $t > t_{heq}$, one has

$$\frac{n_{BH}(t)}{n_{BH}(t_0)} = \left(\frac{t_0}{t_{heq}}\right)^{3/4}\frac{t_{heq}}{t} \qquad (25)$$

Substituting the values of the quantities in Eq.(25) using Eqs.(11-14) and (25), one gets,

$$\dot{\rho}_R + \frac{4\rho_R}{3t} = -\beta B\gamma^{1/4}\frac{\rho(t_0)}{t_0}\left(\frac{t}{t_0}\right)^{B-2} \qquad (26)$$

Using the condition of matter-radiation equality at $t_{heq}$, i.e.,

$$\rho_R(t_{heq}) = \rho_{BH}(t_{heq}) = M(t_{heq})n_{BH}(t_{heq}) \qquad (27)$$

one can solve Eq.(26) to obtain

$$\rho_R(t) \approx \gamma^{-1}\rho(t_0)\left(\frac{t_{heq}}{t}\right) - \frac{\beta B}{B+1/3}\gamma^{1/4}\rho(t_0)\left(\frac{t_0}{t}\right)^{1-B} \qquad (28)$$

where we have neglected a term of higher order in $t_{heq}/t$.

The black hole equation for $t > t_{heq}$ hence gets modified to

$$\dot{M} = B\gamma^{-1}\frac{M}{t_0}\left(\frac{t_{heq}}{t}\right) - C\gamma^{1/4}\frac{M}{t_0}\left(\frac{t_0}{t}\right)^{1-B} \qquad (29)$$

where

$$C = \frac{\beta B}{B+1/3} \qquad (30)$$

Note that we have ignored the evaporation term at this stage since it is even more negligible compared to accretion because of the resultant mass gain up to $t_{heq}$. Using $M(t_{heq}/M_0) = \gamma^B$, the solution of Eq.(29) can be obtained. Beyond $t_{heq}$ the black holes continue to gain mass for a while until the two terms in Eq.(29) become comparable in magnitude. At this stage, the rate of accretion becomes negligible, and one enters the transition regime ($t_t$) after which evaporation takes over the dynamics. Setting the r.h.s of Eq.(29) to 0 at $t = t_t$, one obtains

$$\frac{t_t}{t_{heq}} = \left(\frac{B+1/3}{1-\beta}\right)^{\frac{3}{3\beta+1}} \equiv \delta \qquad (31)$$

Thus the accretion regime lasts for a brief duration in the matter dominated phase, and the maximum black hole mass $M_(t_t) = M_{max}$ at $t_t$ is given from the solution of Eq.(29) by

$$\frac{M(t_t)}{M(t_{heq})} = \exp\left[3B\left(1-\left(\frac{\beta}{B+1/3}\right)^{\frac{1}{3B+1}}\right) + \frac{\beta}{B+1/3}\left(1-\left(\frac{B+1/3}{\beta}\right)^{\frac{3B}{3B+1}}\right)\right] \qquad (32)$$

In the evaporating regime ($t > t_t$), the rate of change of black hole mass is given by the first term in Eq.(6). This can be integrated to give

$$M(t) = \left[M^2(t_t) - \frac{2AM_4^2}{t_c}(t-t_t)\right]^{1/2} \qquad (33)$$

The energy density in radiation $\rho_R(t)$ whose time evolution is governed by Eq.(24) now has a positive contribution from black hole evaporation. Making the appropriate substitutions, one can again integrate $\dot{\rho}_R(t)$ to obtain

$$\rho_R(t) \approx 3\beta\rho(t_0)\gamma^{-3/4}\frac{M(t_t)}{M_0}\left[\delta^{-1} - \frac{t_{heq}}{t}\right] \qquad (34)$$

The universe gets reheated as $\rho_R(t)$ increases with time. The stage of black hole domination lasts up to a time $t_r$ when $\rho_R(t_r) = n_{BH}(t_r)M(t_r)$. Subsequently, radiation domination takes over once again. From Eqs.(33) and (34), one obtains

$$\frac{t_c}{t_r} \approx \frac{3}{2\delta\gamma} + \frac{A}{4\gamma^{2B}}\left(\frac{M_4}{M_0}\right)^2 \qquad (35)$$

The standard low energy cosmology (for $t > t_c$) should emerge as radiation dominated. Otherwise, nucleosynthesis may be problematic due to insufficient reheating and entropy production. We thus demand that the era of black hole domination be over before $t_c$. Requiring $t_r < t_c$, one gets a lower bound on $\beta$ from Eq.(35), i.e.,

$$\beta \geq \left[\frac{4t_0}{3t_c}(B+1/3)^{\frac{3}{3B+1}}\right]^{B+1/4} \qquad (36)$$

The evaporation time of black holes in this scenario can be calculated from Eq.(33). Note that the mass gain between $t_{heq}$ and $t_t$ given by Eq.(32) is negligible if Eq.(36) is satisfied and also $\beta << O(1)$. The black hole lifetime is now given by

$$\frac{t_{end}}{t_4} \approx \left(\frac{M_0}{M_4}\right)^2\frac{t_c}{t_4}\gamma^{2B} \qquad (37)$$

The contribution of accretion in prolonging the lifetime of black holes compared to the modified evaporation effect is given in this scenario by

$$\frac{t_{end}(M,5D)}{t_{end}(M_0,5D)} = \left(\frac{M_{max}}{M_0}\right)^2 \approx \gamma^{2B} \tag{38}$$

Such an enhancement is independent of the value of the initial mass. Thus in this case the lifetime of the whole mass spectrum gets prolonged due to accretion by the same scaling factor $\gamma^{2B}$, as distinct from the scenario of radiation domination throughout where the low mass black holes are affected most. Thus, for instance, taking the value of the AdS radius to be $l/l_4 \simeq 10^{20}$, and $\beta = 10^{-3}$, black holes with $M_0 \simeq 10^{12}$g evaporate during the present era.

To summarize, we have studied the evolution of primordial black holes formed during the radiation dominated high energy phase on the brane. These black holes obey a modified evaporation law as compared to the usual black holes formed in the standard low energy phase. We have shown that the accretion of surrounding radiation completely dominates evaporation as long as radiation domination persists. This results in the net growth of mass of the black holes. Compared to the case of black holes formed with the same initial mass in standard cosmology, the black holes in the braneworld scenario evaporate much later due to the effect of accretion, as well as a modified evaporation law. We have found that the effect of accretion in enhancement of black hole lifetime could be very significant for even low mass black holes for a wide range of parameters. To conclude, it needs to be emphasized that the effect of accretion and its consequences of black holes surviving through key epochs in cosmology [3] need to be studied in more details. Indeed, such investigations have the potential to further constrain the initial energy fraction in black holes vis-a-vis the size of the extra dimension.

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